Decision Support for Family Relocation Decisions under the Section 8 Housing Assistance Program Using Geographic Information Systems and the Analytic Hierarchy Process

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Abstract

Recent evidence suggests that often families seeking housing under the Section 8 program are unable to choose potential destination neighborhoods that maximize their likelihood of successfully exercising housing subsidies and securing beneficial family outcomes. Housing mobility programs such as the Gautreaux Assisted Housing Program and the Moving to Opportunity for Fair Housing Program, which rely on intensive counseling and small cohorts of relocatees, produce beneficial outcomes, however. How could the standard Section 8 program be improved to enable participants to make better location choices?

This article presents a prototype geographic information systems–based decision support system intended to allow individual Section 8 participants, with the assistance of housing authority counselors, to identify potential destination communities that best match their preferences. Users identify attributes of potential destination communities, identify potential destination communities that satisfy certain criteria based on these attributes, rank the attributes according to their individual preferences, and, finally, rank the most-desired destination communities. An example is given using data from metropolitan Pittsburgh.

Keywords: Decision support; Geographic information systems; Housing mobility; Multicriteria decision models

Introduction

In recent years the Section 8 tenant-based housing assistance program has assumed a more prominent role in providing affordable rental housing to low-income families. Often, traditional public housing is located in high-poverty neighborhoods and is of substandard quality. The Section 8 program provides subsidies to families to enable them to acquire higher-quality rental housing in the private market that may more closely match their preferences.

Use of the Section 8 program as a policy tool has been accelerated by a number of related policy initiatives and research results. Federal public housing reform (Housing Opportunity Program Extension Act of 1996) has enabled the demolition of substandard public housing, encouraged a diversification of income groups in existing public housing, and supported the increased use of Section 8 subsidies to provide housing to low-income families. Federal

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welfare reform (Personal Responsibility and Work Opportunity Reconciliation Act of 1996) has placed pressure on low-income families to search for employment or enter job training programs as a requirement for continued assistance, which for the first time has time limits.

Current research has shown an increase in metropolitan suburbanization with no signs yet of a significant decrease in income disparities between suburban areas and central cities (Madden 1999). Ihlanfeldt (1999) argued that a number of factors indicate a spatial mismatch between low-income individuals seeking work and regions enjoying growth in entry-level employment: geographical separation of high-job-growth suburbs from lower-job-growth inner cities and insufficient transportation access to, social networked knowledge about, and housing discrimination within these high-job-growth areas.

Thus, the Section 8 program can be seen as a policy tool not only to enable families to choose better-quality housing consistent with their own preferences, but also to address spatial mismatch directly by facilitating the families' access to entry-level employment. Current research on the Section 8 program, however, indicates that although Section 8 families locate in more desirable areas than the areas in which public housing is found, often they choose areas that are poorer and more racially segregated than typical communities within metropolitan areas (Turner, Popkin, and Cunningham 1999).

One possible cause of this concentration of Section 8 families is that the counseling provided to participating families to help them search for rental housing is not sufficient to enable them to choose communities in which to search that would maximize the probability of better life outcomes. Turner, Popkin, and Cunningham (1999) have shown that the focus of mobility counseling is often on program rules and requirements and less often on identifying potential destination communities whose characteristics can enable better family outcomes. The outcomes realized by Section 8 families, while encouraging overall, are in contrast to the results associated with housing mobility programs with specialized counseling components such as the Gautreaux Assisted Housing Program in the Chicago area (Rosenbaum 1995) and the nationwide Moving to Opportunity for Fair Housing demonstration program (U.S. Department of Housing and Urban Development 1999; Johnson, Ladd, and Ludwig 2001).

Another cause of this concentration is regional barriers to fair housing. Martin and Johnson (1999) argued that in the Pittsburgh region a number of factors impede the access of members of protected classes to housing in their most desired communities. Such factors include evidence of housing discrimination; lack of current real estate data; weak enforcement of existing anti-housing-discrimination laws; and evidence of “futile gesture,” that is, of individuals deciding not to investigate housing opportunities for fear of housing discrimination or of unwelcoming communities. Thus, although improved mobility counseling has the potential to improve life outcomes for all Section 8 families, poor implementation and existing barriers to fair housing inhibit access of program participants to economic opportunity and high-quality services.

The goal of this article is to present a computer-based decision support methodology to enable families in the Section 8 program to make better choices of communities in which to search for private market rental housing. This methodology is represented by a prototype application called housing relocation assistant (HRA), which uses geographic information systems (GIS) to display neighborhood characteristics and select candidate destination communities and a multi-attribute decision methodology called the analytic hierarchy process.
(AHP) to rank user preferences for community attributes and to rank destination communities. This article represents a first step in a larger research agenda: creation of a Web-based application that combines spatial analysis and ranking of destination communities with evaluation of family outcomes associated with use of such a counseling support system in practice.

A brief exposition is given of AHP as it might be applied to housing mobility counseling. A survey of real estate applications of GIS and Web-based relocation tools follows, showing that there is a gap in GIS-based decision support for low-income-housing relocation. Requirements of GIS-based decision support for Section 8 mobility counseling are then provided, followed by a presentation of the preliminary HRA prototype. The article concludes with a summary and identification of issues for further research.

**Analytic Hierarchy Process**

Development of a decision support tool for housing mobility requires specification of two key components: (1) a methodology for eliciting and organizing preferences for community attributes so that selected alternative destination communities can be ranked and (2) a platform with which to display a wide variety of community attributes and to select a set of communities that satisfy certain minimal requirements based on user-selected attributes. This section presents AHP, the multi-attribute decision framework used in this article. It is argued that among alternative decision-theoretic models, AHP is best suited for this article’s target group: inexperienced users with ill-defined preferences who need to make important decisions quickly and easily.

Although a number of researchers have explained the basics of AHP (Ball and Srinivasan 1994; Harker 1987; Saaty 1990, 1994; Zahedi 1986), the presentation by Harker is an excellent combination of principles, theory, and applications and will be the basis for much of the material in this section. AHP, a scientific/logical, as opposed to holistic, approach to solving decision problems, incorporates creative and quantitative analysis of problems. AHP has been used in a wide range of applications, including economics and planning, energy, health, and conflict resolution. What many of these application areas have in common is that there is a qualitative, as well as quantitative, aspect to the problem at hand, and in many cases no method had heretofore existed to incorporate qualitative judgments in a decision-theoretic framework (Zahedi 1986).

AHP depends on three fundamental insights. The first insight of AHP is that any decision problem can be decomposed into a hierarchy consisting of an end goal, criteria, subcriteria, and alternatives. The end goal is the ultimate aim of the decision problem, criteria are attributes by which the quality of the decision made is judged, subcriteria are additional attributes that further describe a particular attribute (subcriteria can be defined in an arbitrarily deep number of levels), and alternatives are different decisions that can be made to achieve the end goal. Figure 1 shows a hierarchy for a hypothetical Section 8 family’s relocation decision.

The figure demonstrates that the ultimate goal of the family’s relocation decision is a ranking of potential destination communities to facilitate a housing search. Potential destination communities that are highly ranked have attribute values that are more closely associated
Figure 1. Sample Hierarchy for Relocation Decisions under the Section 8 Program

End Goal

Criteria

Subcriteria

Alternatives

Rank Alternative Communities for Best Family Outcomes

Crime Rate

Proximity to Employment Opportunities

Distance from Current Community

Educational Quality

Amount of Affordable, Vacant Rental Housing

Central-City Employment

Suburban Employment

Quality of Elementary Education

Quality of Secondary Education

Community 1

Community 2

Community n
with family preferences than are those that are less highly ranked. These rankings are computed using criteria and subcriteria—indicators that characterize the attractiveness of communities along dimensions such as crime rates or educational attainment criteria. The client, with the help of a housing counselor, determines preferences for criteria and subcriteria. Alternatives are specific communities in which the family is considering searching for rental housing.

The second insight of AHP is that generally it is easier for people to estimate values, or preferences, based on paired comparisons on a ratio scale than it is for them to state ordinal amounts. For example, it is easier to estimate distances between Pittsburgh and various other cities through pairwise reciprocal comparisons, than it would be to simply estimate distances in miles (e.g., if Boston is twice as far from Pittsburgh as is New York City, then New York City is one-half as far from Pittsburgh as is Boston). A set of paired comparisons between all alternatives is then created; each paired comparison is represented as coefficients $a_{ij}$, where $i$ and $j$ denote elements of a choice set $C$ (alternative cities). The matrix of paired comparisons is denoted $A = [a_{ij}]$, $i, j = 1, \ldots, n$, where $n$ is the number of elements in the choice set. The $a_{ij}$ values are usually assumed to vary between fixed lower and upper bounds and may be associated with verbal descriptions (Zahedi 1986). For example, alternative 1, which is “strongly preferred” to alternative 2, can be defined as a pairwise weight of nine for alternative 1 compared with alternative 2 (or a pairwise weight of one-ninth for alternative 2 compared with alternative 1). An illustration follows: On the basis of figure 1, a user would be prompted to state the extent to which elementary education quality is more important than secondary education quality in assessing the overall quality of education in a community.

The third insight is that through pairwise comparisons of the desirability of criteria on a given level (or alternatives) with respect to a particular criterion on the next higher level (or the end goal), it is possible to rank all alternatives in such a way that the alternative that best achieves the end goal is identified. This process is done from the bottom up by computing the priorities of all alternatives with respect to each of the criteria (or subcriteria) on the next higher level, then computing the priorities of all criteria (or subcriteria) with respect to the end goal (or criteria) on the next higher level, until all levels have been analyzed. Then, the composite priorities of all alternatives are computed. The result of this analysis is the ranking of all alternatives according to all criteria and subcriteria, denoted by the vector $W = [w_1, w_2, \ldots, w_n]$, where $w_i \in [0, 1]$ is the level of preference for alternative $i$, and $\sum_{i=1}^{n} w_i = 1$. Details of the AHP calculations are shown in the appendix.

A number of extensions to the AHP are of practical as well as theoretical interest. First, if a person is unable to specify a single value for the judgment $a_{ij}$ but instead specifies a range of values (e.g., depending on other attributes, education could range between “important” and “crucial” when a community is being chosen), a distribution of the alternatives’ weights vector $W$ can be generated.

Second, although $\frac{n(n-1)}{2}$ pairwise comparisons are necessary for a set of $n$ attributes or alternatives, this potentially large number of judgments by the decision maker may be reduced through the method of incomplete pairwise comparison (see Harker 1987 for details). Finally, AHP may be used to derive a variety of insights into a decision problem. For example, AHP is normally used to rank alternatives using judgments of relative preferences.
based on pairwise comparisons. Alternatively, AHP may be used to determine matrix $A$ of pairwise comparisons associated with a given decision (e.g., given that a family wishes to search first in community X, then in community Y, the set of attribute comparisons that could give rise to this ranking of alternatives can be derived; see DeTurck 1987 for details).

It is useful to determine how the AHP compares with other multi-attribute decision theoretic techniques for individuals. This discussion will begin with multi-attribute utility theory (MAUT). Whereas the fundamental calculation in the AHP is ratio estimation as presented above, the fundamental calculation in MAUT is the assessment of a utility function by comparing pairs of certain payoffs and probabilistic lotteries (after certain key MAUT assumptions are satisfied; see Keeney and Raiffa 1976 and von Winterfeldt and Edwards 1986 for details). In many contexts it may be significantly easier for a decision maker to estimate ratios than it is to calculate utility functions using probability estimations (but see Keeney and von Winterfeldt 1990 for a counterexample). In addition, MAUT requires that decision makers always be consistent in their judgments, whereas AHP deals formally with inconsistent judgments (Vargas 1987). In particular, it is not necessary that preferences satisfy ordinal transitivity, that is, if $O_i$ denotes “alternative (or attribute) i,” and $\succ$ denotes “preferred to,” it is not necessarily the case that $O_i \succ O_j$ and $O_j \succ O_k$ imply $O_i \succ O_k$. However, given the trade-off matrix $A$, the weights vector $W$ may be computed according to the iterative eigenvector algorithm of Zahedi (1986) to generate a consistency ratio that represents the extent of these inconsistent judgments. Consistency ratios above a given threshold indicate that the decision maker may wish to reexamine trade-off matrix $A$.

Keeney (1992) presented a new decision-making technique called value-focused thinking. Value-focused thinking is a tool by which ends and means objectives may be distinguished in difficult decision problems and the set of useful decision alternatives may be extended. However, value-focused thinking does not by itself allow the decision maker to identify preferred alternatives; some sort of ranking technique is still necessary. Value-focused thinking might be used as a way to design a decision hierarchy.

AHP has had limited application to housing choice. Banai-Kashani (1984) used AHP to perform a housing market simulation in which attributes of locations serve as the higher levels of the hierarchy and locations of housing serve as lower levels of the hierarchy. However, the presentation of this model is quite obscure, and the application does not address actual elicitation of subjects’ preferences. Ball and Srinivasan (1994) applied AHP to the problem of consumers’ selecting specific housing units for purchase. A conceptually similar application to low-income-housing search is proposed, with the following two key modifications:

1. Because the universe of alternatives is given by rental housing units, and because imperfect market information will usually make the compilation of an exhaustive list of available rental housing units impossible, the goal of HRA is to choose regions, or communities, within which users will search for rental housing consistent with their preferences. Thus, the set of attributes of interest is derived from community characteristics rather than housing unit characteristics (though the latter could certainly be a part of the analysis if sufficient data are available).

2. GIS technology must be used to provide contextual information to users to enable them to specify attributes and perform pairwise comparisons, rather than to specify attributes a priori.
Because the intended users of the HRA methodology are low-income families in the Section 8 program, along with public housing authority counselors, any decision-theoretic technique employed should be easy to explain and use and the results should be easy to interpret. It is asserted that AHP is a simpler, more intuitive methodology to enable individual decision making than is MAUT or value-focused thinking. AHP explicitly addresses inconsistent preferences and can incorporate qualitative as well as quantitative domain knowledge. AHP has also been applied previously to housing choice (though not in the manner this article proposes) with promising results. Thus, AHP appears to be better suited for housing counseling assistance than other multi-attribute decision-theoretic techniques examined here.

**Real Estate Decision Support for Housing Mobility**

This section presents applications of GIS, relational databases, and the World Wide Web to real estate in general and residential housing choice in particular. It appears that although a wide range of information technology applications to facilitate housing choice exist, none of them provide the combination of spatial context and decision support that HRA is intended to provide.

GIS, a combination of hardware, software, database, and human and physical infrastructure, displays and analyzes spatial data (Dickinson and Calkins 1988). GIS is relevant to this study because the decision alternatives under consideration—alternative locations in which to search for rental housing—are obviously spatial in character, and attributes associated with these alternatives are naturally considered in a spatial context. Thrall (1998) surveyed applications of GIS in the areas of residential brokerage, appraisal, mortgage default tracking, insurance underwriting, regulatory compliance, and market analysis. Of these, the application areas most relevant to this article are residential brokerage and market analysis. Residential brokerage GIS applications rely on proprietary multiple listing service data, which do not include rental properties. However, features of these applications, including virtual tours of properties and availability of community attributes, would be relevant to a GIS-based counseling system for low-income housing. Market analysis, in particular housing demand forecasting, could help housing analysts and counselors better anticipate changes in counseling policy over time.

There are a number of Web-based applications not yet documented in the literature that enable users to select metropolitan areas, regions within metropolitan areas, or specific properties on the basis of community and/or property attributes; they incorporate GIS to some extent. Overall, these applications tend to incorporate GIS (if they do at all) as a way of visualizing regions (or homes) that satisfy certain criteria but not as a way to choose criteria. That is, often a user is required to choose a region in which to search for housing, or is prompted to specify criteria of interest, without the possibility of spatial analysis to determine what regions or criteria might be more or less preferred. Many of these sites perform the “filtering” function, but only one, BestPlaces.net (www.bestplaces.net), actually computes candidate regions sorted according to user preferences. This site requires the user to specify preferences for a number of community attributes, but it does not provide pairwise ranking of attributes, nor does it incorporate GIS, either for attribute ranking or for results display.

Another information technology application area of interest to this work is that of relational database case management tools to assist nonprofit organizations in counseling and track-
ing families seeking affordable housing. The emphasis of all these tools, however, is capturing family characteristics and tracking family outcomes associated with specific counseling strategies, rather than capturing characteristics of destination regions and eliciting family preferences for certain regions or for certain regional characteristics. Thus, although there are a number of current applications that address household mobility and housing case management, they lack many key features especially relevant to Section 8 counseling.

The research on operations research/management science (OR/MS) applications to real estate using GIS is thin. Koch (1999) presented a number of OR/MS applications using GIS in areas such as location, routing, political redistricting, and public health; he made a persuasive case that GIS can improve the quality of decision making that may have previously relied on decision support systems without an explicit spatial component. Johnson (2001) used GIS and integer programming in a spatial decision support system to enable planners to generate alternative allocations of families in subsidized housing over a metropolitan area; the allocations correspond to alternative policy initiatives.

Thus, there is an opportunity to combine GIS, relational databases, and OR/MS models in a novel way to enable elicitation of values, expansion of choice sets, and selection of alternatives at the individual level rather than for aggregate planning.

Architecture of a Spatial Decision Support System for Housing Mobility Assistance

This section contains a description of requirements for HRA. These requirements are designed to address specific needs of low-income families in the Section 8 program who have imperfect knowledge of their preferences and are seeking relocation assistance to improve family outcomes.

HRA is a decision support system (DSS) defined by Peterson (1999) as “an interactive computer-based system that helps decision makers utilize data to solve unstructured problems” (Sprague and Carlson 1982, quoted in Peterson 1999, 137). HRA is composed of a decision model, GIS data, and a user interface for input requirements and output reporting. Because HRA uses spatial data, it may also be classified as a spatial decision support system (SDSS).

Although HRA is intended to help families choose suitable communities in which to search for rental housing, at this point it is not clear what “community” means. Sociologists have developed many alternative definitions of community; the definition used here is that of “a territorially organized population with common ties and social interaction” (Lyon 1989, 6). Thus, a community could be a municipality, school district, township, or city neighborhood. A community could also be defined in a purely ad hoc manner, such as the portion of the local region accessible by mass transit from my current location in 45 minutes or less. Finally, a community can be defined as the area within standard geographic units used for census enumeration, such as tracts or block groups.

The last definition of community is less appealing than the former definitions because people are less likely to identify with fairly abstract concepts such as tracts and block groups than they are with other spatial units. However, units such as tracts are worth considering for the purposes of relocation analysis because certain demographic characteristics such as ethnic/
racial composition may show significant variation between tracts that might be obscured at coarser geographic aggregations.

What kinds of community indicators are desirable for HRA? Two types of local indicators would be ideal: objective indicators, which measure tangible physical quantities (e.g., socioeconomic characteristics or diversity of housing stock), and subjective indicators, which measure attitudes about events (e.g., how welcoming a community is perceived to be toward new neighbors). Although subjective indicators are clearly important in making relocation decisions, they are difficult both to collect for individual communities and to compare across communities. However, many objective indicators are widely (and often freely) available in data sets such as the summary tape file 3A (STF3A) data tape from the U.S. Bureau of the Census.

There are two strategies that may be pursued to incorporate subjective indicators in a DSS such as HRA. The first is to identify objective indicators that serve as plausible proxies for subjective indicators. For example, the level of tolerance a community may have toward new residents of a different race, ethnicity, or economic class could be expressed as the number of allegations of housing discrimination filed in that community in the previous year. Another strategy is to allow the user to simply delete from consideration certain potential destination regions that score poorly on one or more negative subjective indicators or, at the stage of ranking alternative destination communities using AHP, to include in a list of attributes of alternative destination communities one or more subjective criteria not easily mapped.

As an example of the former strategy, a user may simply not consider a portion of a study area because of widespread, but poorly documented, allegations of police misconduct toward minorities. Therefore, in this article only objective indicators will be considered for purposes of spatial data display, though at the stage of choosing or ranking potential destination communities, more qualitative considerations may come to bear.

A number of categories of objective indicators serve as the basis for discussing HRA's data requirements. These categories are designed to be relevant to a Section 8 recipient's housing search (see, e.g., Lyon 1989):

1. Availability of high-quality entry-level employment
   - Level and rate of appreciation in entry-level/low-skill occupations
   - Recorded complaints of employment discrimination

2. Availability of affordable rental housing
   - Vacancy levels in affordable rental housing
   - Average lease-up rates for families using Section 8 subsidies
   - Variation in fair market rents across the study area

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1 The STF3A data set produced by the U.S. Census Bureau contains a wide range of population and housing items in geographies that range from the state through the census tract to the block group (U.S. Bureau of the Census 2001).

2 This strategy is undesirable because of the increased likelihood of highly inconsistent judgments.
3. Public transit accessibility to jobs
   • Areas of high growth in entry-level employment served by public transit from higher-poverty areas
   • Commute times between higher-poverty areas and areas of high job growth

4. Social services support
   • Proximity to child care, job training, and food banks
   • Fraction of population with characteristics indicative of use of social services (e.g., single-female headed, below poverty line, unemployed)

5. Quality of education
   • Per-pupil expenditures
   • Student education attainment
   • Classroom size
   • Dropout and retention rates
   • Quality of teacher training

6. Public safety
   • Violent and nonviolent crime rates
   • Accessibility to fire and emergency medical services
   • Recorded complaints of police misconduct

7. Local amenities and demographic characteristics
   • Housing quality (residential crowding, percentage of housing units with structural defects)
   • Trends in property value appreciation
   • Recorded complaints of housing discrimination
   • Proximity to parks, zoos, and other cultural amenities
   • Presence of nonprofit and political organizations focused on needs of low-income families
   • Portions of the population classified by age, gender, race, ethnicity, disability status, income, and family size

8. Quality of life
   • Infant mortality
   • Percentage of families that lack medical insurance
   • Proximity to waste dumps and sites known to represent health hazards
   • Fraction of housing stock that is abandoned
   • Ambient noise levels

Because clients and counselors may have different conceptions of ideal indicators, it is useful to divide this wide range of quantitative indicators into those considered ideal from the client’s point of view as opposed to those considered ideal from the counselor’s point of view. For example, although participants at the Boston site of the Moving to Opportunity mobility experiment listed fear of crime as a primary motivation, relatively few families listed access to employment (Katz, Kling, and Liebman 2001). As described later in this section and in the one to follow, a primary challenge for counselors using HRA will be to help clients become aware of and consider indicators of neighborhood quality that may not be of primary inter-
est to them, at least initially, but which may, on the basis of social science research, be most closely associated with beneficial family outcomes.

Many of the objective indicators previously listed are widely available from local, state, and federal sources. Demographic data may be obtained from census data sets; private vendors such as Claritas furnish trended census data that may provide a timelier estimate of current objective measures of interest. Employment levels may be obtained from the census transportation planning package; levels of job growth may be obtained from ES 202 data tabulated by the U.S. Bureau of Labor Statistics.\(^3\) Measures of education quality may be obtained from state education departments or local school districts, and measures of education attainment are available from census tabulations. Health outcomes can be obtained from state and county health departments. Public safety data are available from state organizations such as the state police or justice departments. Tabulations of discrimination complaints are available from the U.S. Department of Housing and Urban Development (housing), the U.S. Department of Labor (employment), as well as from many local government and nongovernment agencies. Transportation data are tabulated in various ways. Census STF3A data include commute times based on residence; census transportation planning package data record commute times based on employer location. Local transit authorities may have point-to-point transit time estimates. Finally, a variety of state and local agencies and nonprofits may tabulate locations of social service providers.

As indicated above, information on local rental real estate markets helps counselors provide effective mobility counseling. Unfortunately, although detailed data on owner-occupied housing are available from local governments and various commercial data providers, that is not true for rental housing. Some cities have organizations that perform rental market surveys, the American Housing Survey generates information periodically on individual units based on a sample of a metropolitan area’s households, and the decennial census contains counts of occupied and vacant rental units and their asking prices. It is rare, however, to find a source of current data on rental real estate markets across an entire metropolitan area. This is true in particular for the metropolitan Pittsburgh region, which serves as a test bed for HRA.\(^4\)

HRA should be able to display all relevant local indicators in GIS. That is, a user should be able to construct thematic or dot-density maps,\(^5\) which show variations in any chosen indicator across the study area and are based on lists of available themes (dot-density or thematic maps) that the user selects or deselects as necessary. In addition, the user should be able to perform certain basic geographic functions: zoom in or out, pan, search, and perform spatial queries and boundary selection. Spatial queries could be performed via a dialog box in which the user selects the following: a boundary file (e.g., municipalities), one or more variables

\(^3\) The ES 202 program, formally known as the Covered Employment and Wages program, collects employment and wage data for civilian workers covered under state or federal unemployment insurance programs (U.S. Bureau of Labor Statistics 1997).

\(^4\) Some agencies providing housing assistance may collect data on landlords who are known to provide affordable housing. These incomplete data may serve as a proxy for availability of affordable housing, though with a bias toward areas in which low-income families have traditionally resided.

\(^5\) Thematic maps display certain geographic boundaries shaded a variety of colors corresponding to actual values or ranges of values of a particular variable. Dot-density maps display dots, circles, or other geometric forms associated with certain geographic regions; their size varies corresponding to actual values or ranges of values of a particular variable.
whose values will define the scope of records selected, and the permissible range of values for each variable. The output of this process is a new view that contains a boundary file corresponding to the geographic units.

A sample illustration of this process using standard GIS functionality in ArcView 3.2 (Environmental Sciences Research Institute [ESRI] 2000) is given in figure 2, which highlights municipalities in Allegheny County (PA) that have 6 or fewer housing complaints in 1999 and whose 1990 violent crime rate is 8 crimes per 1,000 people, or lower.6

Boundary selection is done with the mouse and consists of the user physically selecting (possibly noncontiguous) geographic units with the mouse, then choosing a toolbar option to create a new view in which the selected geographic units compose a new theme. Both spatial queries and boundary selection will be used to define a set of potential destination communities that will be ranked on the basis of user preferences.

ArcView provides direct access to spatial data and a wide variety of spatial analytic functionality, both built-in and extensible via the proprietary scripting language, Avenue. Although a more robust system could be developed using a custom interface such as Microsoft Visual Basic, combined with modules to enable display and analysis of spatial data such as ESRI’s MapObjects (see, e.g., Johnson 2001), the HRA prototype shown in this article uses ArcView GIS alone to carry out spatial data display and analysis.

Ranking of alternative destinations is done using AHP. Expert Choice (Expert Choice, Inc. 1999) is a widely available implementation of AHP that contains an intuitive graphical user interface to allow users to specify criteria and goals, either directly or via displays that allow them to brainstorm and refine criteria and goal categories. Moreover, pairwise comparison of criteria and goals can be done via manipulation of bar charts, verbal scales, or direct numeric comparisons in matrices. Finally, Expert Choice has a robust sensitivity analysis module that allows the user to determine how rankings of goals are affected by changes in preferences for various criteria. Currently, Expert Choice is available only as a stand-alone application, and cannot easily be integrated into custom applications. Despite this significant limitation, HRA uses Expert Choice for multicriteria decision analysis.

Ideally, HRA would be available on the World Wide Web rather than as a desktop application. Making HRA available as a Web application minimizes client hardware and software requirements (only a modem and a Web browser are necessary), minimizes application maintenance overhead (only a single modification to an application residing on a Web server is necessary), and maximizes the availability of the application to spatially dispersed clients. However, given limitations in the architecture of Expert Choice, the HRA prototype is available only for single-user desktop computing.

The role of the housing counselor is key to the successful use of HRA. The housing counselor will work as a facilitator to the Section 8 family, prodding the family to consider communities and community indicators that are different from those the family may be used to. The counselor must be skilled in the use of GIS and AHP and must be able to view spatial data in multiple ways and perform spatial queries that reflect family preferences. In addition, the

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6 Here, and subsequently, maps illustrating characteristics of Allegheny County omit the city of Pittsburgh, the municipality of Penn Hills, and the city of McKeesport, corresponding to data used in Martin and Johnson (1999).
Figure 2. Spatial Query in ArcView: Municipalities with Six or Fewer Housing Complaints in 1999 and a 1990 Crime Rate of Eight Crimes per 1,000 People, or Less (excluding Pittsburgh, Penn Hills, and McKeesport).

Note: This figure was created using ArcView 3.2 software.
counselor must be able to synthesize possibly diffuse or contradictory family preferences into a hierarchy with pairwise comparisons suitable for analysis and to facilitate sensitivity analysis that enables family members to revise their judgments.

Finally, HRA ought to be integrated with the other relational database applications, presented above, that perform client tracking for housing organizations. That is key for evaluation purposes. It is of great interest to know how family preferences for communities, and community attributes and family, are linked to actual initial and subsequent moves under the Section 8 program.

Prototype SDSS for Housing Relocation: HRA

A preliminary prototype of HRA uses spatial data for the Pittsburgh metropolitan area that were compiled by Martin and Johnson (1999) for an analysis of impediments to fair housing in suburban Allegheny County. In practice, a DSS such as HRA should use data for both city and suburban areas to ensure the widest range of destination communities. The key functionality of HRA, however, is independent of the composition of the study area, and HRA can be easily extended to address spatial data for the city of Pittsburgh as well as its surrounding suburbs.

Table 1 contains a list of each data element, the unit of geographic detail corresponding to each element, and the source of the data. Data elements whose geographic detail is classified as “point” display on a map as points, but they may be aggregated to any desired extant (tract, municipality, etc.) via geocoding. Data elements whose geographic detail is classified as “line segment” display on a map as line segments and cannot be aggregated to geographic extants in any meaningful way. Data elements whose geographic detail is classified as “polygon” may cross standard geographic boundaries such as census tracts or ZIP codes and, in these cases, cannot be easily aggregated to other standard geographic units.

Note that table 1 has no measure of availability of affordable rental housing; no current measure of rental vacancies was available aside from data from the 1990 census. Given changes in real estate markets over time, it was decided that including this data element could be misleading. An example of the data listed in table 1 is shown in figure 3, which presents a thematic map of ZIP code regions according to the number of high-paying service establishments located within them. It is straightforward to illustrate other transportation-related data, such as the portion of the study area accessible by mass transit within a certain time standard from a given origin.

A number of spatial queries to generate information of use to a client may be performed using spatial data such as those illustrated above. For example, a person could choose all municipalities with 1990 violent crime rates lower than 1 crime per 1,000 people that are (a) partially or fully contained in ZIP codes with at least 100 service establishments and (b) partially or fully contained in a polygon indicating a 60-minute travel time via mass transit from the high-poverty municipality of Braddock.

The HRA prototype uses ArcView 3.2 alone for all spatial display and analysis without—at this stage—Avenue scripts to automate some common processes. In particular, the list of cri-
Table 1. Spatial Data Contained in HRA

<table>
<thead>
<tr>
<th>Data Element</th>
<th>Geographic Detail</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990 violent crime rate (per 1,000 people)</td>
<td>Municipality</td>
<td>Pennsylvania State Police (1990)</td>
</tr>
<tr>
<td>Number of hazardous waste sites (Superfund Archive, Priority, and Nonpriority)</td>
<td>Municipality</td>
<td>U.S. Environmental Protection Agency (1999); Pennsylvania Department of Environmental Protection (1999)</td>
</tr>
<tr>
<td>Number of complaints alleging housing discrimination</td>
<td>Municipality</td>
<td>Fair Housing Partnership of Greater Pittsburgh (1999); Pennsylvania Human Relations Commission (1999)</td>
</tr>
<tr>
<td>Establishment counts by sector (overall, construction, financial, manufacturing, mining, retail, service, high-paying service, low-paying service, transportation, unclassified)</td>
<td>ZIP code</td>
<td>U.S. Bureau of the Census (1997)</td>
</tr>
<tr>
<td>Education measures (5th-, 8th-, 11th-grade math and reading achievement outcomes, student-teacher ratios, teacher education levels)</td>
<td>School district</td>
<td>Education Policy Issues Center (1999)</td>
</tr>
<tr>
<td>Subsidized housing (Section 8, public housing)</td>
<td>Municipality, tract</td>
<td>Allegheny County Housing Authority (1999)</td>
</tr>
<tr>
<td>Total number of zoning restrictions</td>
<td>Municipality</td>
<td>Allegheny County Department of Economic Development (1999)</td>
</tr>
<tr>
<td>Churches</td>
<td>Point</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Primary/secondary schools</td>
<td>Point</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Adult education facilities</td>
<td>Point</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Landmarks</td>
<td>Point</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Day care facilities</td>
<td>Point</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Businesses</td>
<td>Point</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Parks</td>
<td>Polygon (multiple)</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Roads</td>
<td>Line segment</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Bus routes</td>
<td>Line segment</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
<tr>
<td>Public transit access regions: 15 min., 30 min., 45 min., 60 min. (various high-poverty communities in suburban Allegheny County)</td>
<td>Polygon (single)</td>
<td>Regional Planning Technologies (1999)</td>
</tr>
</tbody>
</table>
Figure 3. High-Paying Service Establishment Counts by ZIP Code

Note: This figure was created using ArcView 3.2 software.
Criteria based on available spatial data is recorded manually. Also, producing a list of candidate regions as output from a spatial query and/or a boundary selection requires that the municipalities selected be manually saved as a new shapefile and that the list of municipalities be extracted to a dBase-format file through the Summarize command. Expert Choice Professional, Version 9.5, is used to structure hierarchies, rank alternatives, and perform sensitivity analysis. Expert Choice is not linked directly to ArcView.

To demonstrate the operation of the HRA prototype, let us assume that a family considering relocation from the high-poverty community of McKees Rocks to other, more desirable communities in suburban Allegheny County is using a Section 8 subsidy. Assume also that, with the assistance of a relocation counselor, the family has examined the entire set of spatial data listed in table 1 using ArcView’s standard mapping techniques.

To select potential destination communities, the family must specify a set of criteria by which communities may be compared. It is proposed that the housing counselor elicit criteria by asking the family a series of wide-ranging questions. Such questions could include the following: What characteristics of a neighborhood are most important to you? What is the main reason you wish to move from your current neighborhood? What would an ideal neighborhood look like to you? Note that it is not important that the family specify, at the outset, ranges of values for the criteria that are important to them. Presumably, spatial analysis with GIS will suggest to families ranges of certain criteria that distinguish more-desirable from less-desirable destination communities.

Let us assume that such a “framing” discussion has been held and that the family has identified the following criteria as key to their relocation decision:

1. School quality
2. Crime rate
3. Proximity to family and social networks in McKees Rocks
4. Proximity to employment opportunities in the retail sector

Assume as well that, using these criteria, the family has examined displays of spatial data similar to those contained in figures 2 and 3. On the basis of the family’s assessment of spatial variations in these criteria and information from the counselor regarding realistic target levels for these criteria, the family has specified the following minimum requirements for any destination community:

1. The school district of which it is a part must have at least 15 percent of its 5th graders scoring in the 75th percentile or higher in the state mathematics assessment test.
2. Its 1990 violent crime rate must not exceed 1 crime per 1,000 people.
3. It must be within a one-hour bus ride of McKees Rocks.
4. Any ZIP code that contains it must have at least 100 retail establishments.
Figures 4 to 6 show the criteria listed above. The spatial analysis in ArcView that implements these preferences consists of spatial queries and spatial intersections to select the regions (municipalities, ZIP codes, school districts) that satisfy these criteria. For this example, it is assumed that the preferred geographic unit of analysis is the municipality. Forty-four municipalities in suburban Allegheny County have their center in the polygon defined by a 60-minute mass transit travel distance from McKees Rocks (figure 4). However, only 12 of these municipalities have 1990 violent crime rates that do not exceed 1 crime per 1,000 people (figure 5). A similar spatial analysis finds that only 7 of these 12 municipalities have 100 or more retail establishments (not shown). However, these same 7 municipalities are part of school districts in which 15 percent or more of fifth graders excel on the math achievement test (figure 6). The municipalities that satisfy all these criteria—Baldwin, Ingram, Moon, Mount Lebanon, Osborne, Ross, and Whitehall—along with McKees Rocks, are shown in figure 7.

At this point the data needed to begin AHP are available. The goal is defined as follows: “Choose a municipality in which to search for rental housing using a Section 8 subsidy that maximizes the family’s chances of a beneficial life outcome.” The criteria, residing on a single level, are school quality, crime, family proximity, and employment proximity. The alternatives, derived from the spatial analysis above, are the 7 municipalities listed above. The Expert Choice user interface makes the representation of this hierarchy into data structures possible, allowing the user to view definitions of criteria and alternatives, to save and print problem hierarchies, and to assess preferences for criteria and alternatives in a variety of ways. Initially, Expert Choice assigns equal preferences to each of the four criteria and seven alternatives.

The user first performs pairwise comparisons of each alternative with respect to each of the criteria, using expressions of strength of preference to show how much one alternative is preferred to another. A typical method of pairwise comparison uses verbal preference measures, for example, “With respect to the attribute of crime, Ingram municipality is strongly to very strongly preferred to Baldwin municipality.” This method of preference elicitation appears to be most appropriate for inexperienced users such as Section 8 families, as opposed to alternative modes of pairwise comparison, such as pie charts or direct numerical elicitation.

After hypothetical pairwise comparisons for criteria were entered, Expert Choice determined that consistency ratios for each criterion (see the appendix for a definition) computed in the Priorities module were high relative to a desirable threshold value of 0.10 (Saaty 1994): 0.32 for crime, 0.26 for education, 0.3 for proximity to family, and 0.19 for proximity to employment. Although high consistency indices are not necessarily an indication of irrational or biased preferences, in practice these values would be a cue for the counselor and family to review the pairwise comparison process, ensuring, for example, that $X$ and $Y$, two attributes that have sim-

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7 To perform the spatial intersection, ArcView provides a variety of choices governing the relationships of two themes. It is assumed that two polygons intersect if their centers are contained in the intersection (the choice “Have Their Center In”), concluding that features should be reasonably accessible, as measured by transit access to the center of the feature, whereas “Intersect” throws the net too widely, including regions that only marginally intersect the 60-minute mass transit accessibility region.

8 Although Expert Choice contains a Structuring module allowing the user to clarify criteria, for this example it is assumed that this process has already been completed through the steps mentioned above.
Figure 4. Portion of Allegheny County Accessible by Mass Transit within 60 Minutes from McKees Rocks

Note: This figure was created using ArcView 3.2 software.
Figure 5. Distribution of 1990 Violent Crime Rates in Allegheny County, by Municipality

Note: This figure was created using ArcView 3.2 software.
Figure 6. Distribution of Percentages of Fifth Graders in Highest Percentile on Standardized Math Test, by School District

*Note:* This figure was created using ArcView 3.2 software.
Figure 7. Allegheny County Municipalities that Satisfy Criteria of Hypothetical User

Note: This figure was created using ArcView 3.2 software.
ilar relationships to another criterion $Z$, are in fact given similar pairwise comparison values with respect to $Z$.

After completing the pairwise comparison process, Expert Choice computes the strength of preference values for each of the alternatives with respect to each of the criteria, then for each of the alternatives alone by solving equation A.2 (see the appendix). Table 2 presents aggregate strength of preference values for the alternatives with respect to the criteria, and figure 8 shows the summary screen from the analysis process.

Table 2. Aggregate Strength of Preference Results: Criteria and Alternatives with Respect to Criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Crime</th>
<th>Education</th>
<th>Proximity to Employment</th>
<th>Proximity to Family</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baldwin</td>
<td>0.069</td>
<td>0.102</td>
<td>0.101</td>
<td>0.072</td>
</tr>
<tr>
<td>Ingram</td>
<td>0.136</td>
<td>0.088</td>
<td>0.034</td>
<td>0.238</td>
</tr>
<tr>
<td>Moon</td>
<td>0.143</td>
<td>0.210</td>
<td>0.236</td>
<td>0.263</td>
</tr>
<tr>
<td>Mount Lebanon</td>
<td>0.294</td>
<td>0.254</td>
<td>0.078</td>
<td>0.087</td>
</tr>
<tr>
<td>Osborne</td>
<td>0.044</td>
<td>0.132</td>
<td>0.085</td>
<td>0.171</td>
</tr>
<tr>
<td>Ross</td>
<td>0.268</td>
<td>0.181</td>
<td>0.330</td>
<td>0.132</td>
</tr>
<tr>
<td>Whitehall</td>
<td>0.045</td>
<td>0.033</td>
<td>0.135</td>
<td>0.037</td>
</tr>
<tr>
<td>Overall strength of preference</td>
<td>0.152</td>
<td>0.426</td>
<td>0.359</td>
<td>0.063</td>
</tr>
</tbody>
</table>

Note: Data displayed show the outcomes of the first two steps of the three-step AHP. In step 1, weights, or strength of preference values for each attribute, are computed without respect to the alternatives under consideration. The last row of the table indicates that education is the most highly preferred attribute of a potential destination community, followed by proximity to employment. In step 2, the weights for all alternatives with respect to each criterion are computed. The figures in bold in each attribute column above the last row indicate the alternative that is most preferred with respect to the given criterion.

Table 2 indicates that overall strength of preference for all alternatives is fairly diffuse. The figures in bold in each column of criteria indicate the alternative that is most preferred with respect to the given criterion. For example, Mount Lebanon is most preferred with respect to crime and education (with Ross and Moon, respectively, trailing not far behind); Ross is somewhat more strongly preferred with respect to proximity to employment, with Moon substantially less preferred; and Moon is most preferred with respect to proximity to family, with Ingram trailing close behind. Pairwise comparisons of the criteria themselves with respect to the end goal shows that in this example education is most important for choosing a new community, with proximity to employment somewhat farther behind, and proximity to family not very important at all.

Figure 8 shows that Ross is weakly more preferred than all other municipalities, with Moon and Mount Lebanon not far behind. Other communities, more distant from McKees Rocks and somewhat less affluent than Ross, Moon, and Mount Lebanon, are clustered at the lower end of the preference scale. Recall that the interpretation of this solution is that the Section 8 family should concentrate its housing search first in Ross, then in Moon, and so on.

The overall inconsistency ratio, which is 0.17, was computed essentially by weighting the inconsistency ratios for each criterion by the computed priority for each criterion and adding overall criteria, then dividing by a number that represents a weighted consistency value if
Figure 8. Rankings of Potential Destination Communities Produced by AHP

Note: This figure was created using Expert Choice Professional 2000 software.
random values were entered in the pairwise comparison matrix $A$. This value, which exceeds the threshold of 0.10 presented above, reinforces our intuition that the pairwise comparisons should be reviewed to minimize the extent of serious inconsistencies in the judgment of the hypothetical client.

The fact that the preference gap between Ross and Moon is fairly small is a cue that sensitivity analysis might yield interesting insights into changes in orderings of alternative communities associated with changes in preferences. For that purpose, figure 9, a sensitivity graph, is shown. The graph is one of four kinds of graphical displays Expert Choice offers to examine characteristics of solutions to AHP models. Here, the x-axis values represent each of the criteria, with the right-most x-axis value representing overall preferences for all criteria. Bars at each x-axis value represent overall strength of preference for each criterion. Horizontal lines represent the user's overall preference for each alternative community with respect to the criterion at which the horizontal line crosses the vertical axis. Aggregate strength-of-preference values for each criterion can be manipulated by dragging each bar up or down; in response, the height and slope of the horizontal lines change, showing how overall preferences for alternatives change given changes in preferences for criteria.

This figure shows that if proximity to family were preferred much more strongly overall than any other criteria, the rank order of communities would change, with Moon being most preferred and Ingram being next most preferred. This sensitivity graph could be manipulated in many ways, encouraging families to examine their preferences closely and, conversely, determining which preferences are associated with a desire to live in a particular community, independent of pairwise comparisons. In this way family members can judge for themselves which factors are really important in their decision making.

**Conclusion and Next Steps**

This article has demonstrated that an information technology approach to client relocation counseling for the Section 8 program is justified by current social science results in housing mobility and spatial mismatch. Also, it is argued that such a methodology, based on GIS, OR/MS, and relational databases, can be implemented in a prototype system using existing software tools. This methodology can be used to assess client preferences for various community attributes and to rank potential destination communities in which clients would search for rental housing with Section 8 subsidies. This methodology, however, is hampered by (1) the lack of easily available rental market data that would allow the decision process to incorporate availability of rental housing and (2) the difficulty in navigating between two physically separate applications.

The first drawback could be addressed in part by noting that many housing counselors have compiled lists of preferred landlords—property owners known to be open to renting to Section 8 families. These lists could be geocoded, displayed in the GIS, and used as criteria for selecting destination communities. As noted previously, such lists are subject to bias inasmuch as they represent areas to which families have historically migrated, as opposed to potentially desirable, but less well known areas.

The second drawback will be more difficult to remedy. Johnson (2001) has used the Visual Basic development environment with ESRI's MapObjects to provide GIS functionality with-
Figure 9. Sensitivity Analysis for the Housing Search Problem Using AHP

Note: This figure was created using Expert Choice Professional 2000 software.
in an application containing features quite specific to the planning problem at hand (in this case, aggregate planning via alternatives generation for the Section 8 program over a large metropolitan area). On the basis of the successful results reported in Johnson's article, it would appear that such an approach might be beneficial here if a way could be found to bind Expert Choice more closely to applications that might invoke it. It is argued that such an application could eventually be Web-enabled to ease hardware and software infrastructure requirements at public housing authorities.

It is crucial that the practical utility of this counseling approach be verified through the on-site testing of a robust implementation of this application in a local public housing authority with actual Section 8 families. Such an evaluation could be done by comparing the outcomes of families provided counseling with an assistance tool such as HRA to the outcomes of families counseled without it. This topic is a subject of ongoing research.

Appendix

Details of Analytic Hierarchy Process Calculations

The analytic hierarchy process (AHP) is based on the following four simple axioms that govern the calculations that rank alternatives according to strength of preference for their attributes.

Axiom 1. Given any two alternatives (or subcriteria) \( i \) and \( j \) out of the set of alternatives \( A \), the decision maker is able to provide a pairwise comparison \( a_{ij} \) of these alternatives under any criterion \( c \) from the set of criteria \( C \) on a ratio scale that is reciprocal; that is, \( a_{ji} = 1/a_{ij} \) for all \( i, j \in A \).

Axiom 2. When comparing any two alternatives \( i, j \in A \), the decision maker never judges one to be infinitely better than any other under any criterion \( c \in C \); that is, \( a_{ij} \neq \infty \) for all \( i, j \in A \).

Axiom 3. The decision problem can be formulated as a hierarchy.

Axiom 4. All criteria and alternatives that affect the given decision problem are represented in the hierarchy. That is, all the decision maker's intuition must be represented (or excluded) in terms of criteria and alternatives in the structure and must be assigned priorities that are compatible with the intuition.

These axioms have a number of important implications. First, from axiom 2, we exclude the possibility that the user has an infinite preference for one criterion; if that were so, the user would simply choose the alternative with the most favorable value according to that criterion. Second, from axiom 3, if an end goal, criteria, subcriteria, and alternatives can be identified, a hierarchy can be formulated (though some extra work may be necessary). Third, from axiom 4, even though the AHP suffers from a weakness (primarily of theoretical concern) called rank reversal, this problem may be avoided if all alternatives and criteria that mat-

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9 Rank reversal is the phenomenon in which the inclusion of irrelevant alternatives can change the alternatives rankings. For example, if the goal is to choose between models of cars to purchase, and if two alternative hierarchies are specified, one including model X and another including in addition a model Y, which is an exact substitute for model X, rank orderings of car models in the two hierarchies may be different, even if in practical terms the sets of alternatives for the hierarchies are identical.
ter are included in the hierarchy. Fourth, the reliance on judgment in the calculation of weights for alternatives and criteria implies that assessments may be inconsistent; the AHP accounts for this possibility by calculating a consistency ratio that represents the probability that the entries in a trade-off matrix were filled in at random. Consistency ratios above a given threshold (usually 0.1) are a signal that decision makers should reexamine their ratio trade-offs.

The relative weights of $n$ elements at one stage in the hierarchy are calculated in the following way (Zahedi 1986): Assume that each pairwise comparison $a_{ij}$ is expressed as the ratio of arbitrary weights $w_i/w_j$ and that the pairwise comparison matrix $A$ is given as $A = \begin{bmatrix} w_1/w_1 & \cdots & w_1/w_n \\ \vdots & \ddots & \vdots \\ w_n/w_1 & \cdots & w_n/w_n \end{bmatrix}$. If this pairwise comparison matrix were completely consistent, the set of priorities for the given elements would be given by the vector $W = [w_1, w_2, \ldots, w_n]$ and the following matrix equation would hold:

$$ A \cdot W^T = n \cdot W^T, \quad (A.1) $$

where $T$ represents the matrix transpose.

However, in practice matrix $A$ contains inconsistencies. To calculate $A$ given inconsistent judgments, it is necessary to solve the matrix equation

$$ \hat{A} \cdot \hat{W} = \lambda_{\text{max}} \cdot \hat{W}, \quad (A.2) $$

where $\hat{A}$ and $\hat{W}$ are estimates of the true quantities $A$ and $W$ given by the decision maker. It can be shown that $\lambda_{\text{max}}$ is the largest eigenvalue of $\hat{A}$, and $\hat{W}$ is the right eigenvector of $\hat{A}$. The technique most often used to calculate $\hat{A}$ and $\lambda_{\text{max}}$ is an iterative method developed by Saaty (1990) and used in the AHP software Expert Choice (Expert Choice, Inc. 1999).

References


Allegheny County Housing Authority. 1999. Electronic data set of public housing and Section 8–assisted families. Pittsburgh.


